Study of the Trend Prediction Method Based on Max Lyapunov Exponent for Rotating Machine Sets

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Abstract:. Develop condition prediction is very important for sets' safe operation. Due to the nonlinear and Non-Stationary running condition of the equipment，max Lyapunov exponent is introduced to the fault trend prediction of large rotating mechanical sets based on chaos theory. Two methods of proposing $\hat{f}$ and $\hat{F}$ are discussed and the arithmetic of max prediction time of chaos time series is provided. Experiment data analysis of large rotating machine shows that this method has excellent performance for condition trend prediction.

Key word: Max Lyapunov exponent，Large rotating machine sets，Developing condition prediction

1．General statement

Mechanical equipment's status prediction belongs to indispensable link in equipment fault diagnosis. Rotating machines are applied with widest range in industry and they generally run in high velocity, which make the status prediction very important. Equipment's status prediction is based upon historic data acquired from continuous monitoring of equipment so as to confirm current operating status and predict its developing trend and remaining service life, which is very significant for equipment maintenance and final decision of equipment repairing.

Traditional prediction methods include dynamics method and mathematical statistics method. Both of the above-mentioned two methods have common features, namely, they establish data model at first and then execute computation and prediction according to the data model, which makes subjectivity unavoidable; at the same time, variable non-linear factors exist and nonlinearity has become inherent characteristic of mechanical system with the appearance of chaotic motion. Therefore, generally speaking, the collected system vibration time series signals are also nonlinear, which make it hard to use mathematic formula to describe it in an accurate way; in comparison with linear time series, nonlinear time series contains wider and deeper contents for study, which is commonly represented by richer and more complicated motion phenomena and even the simple nonlinear time series can also contain rich characteristic information and very complicated features. It is hard for the adoption of traditional method to predict its status, which cannot guarantee its accuracy and credibility. Therefore, the study of system chaos time series prediction is very significant.

The so-called time series prediction is actually to estimate the value x_{n+k} at moment n + k(k > 0) in future based upon historic observed values of time series $x_1,x_2,\ldots\ldots,x_n$. In the field of mechanical status monitoring and fault diagnosis, time series prediction of vibration signals contains very important values, which can be used to monitor system status, abnormal behaviors and predict developing trend of faults, etc.

2. Chaos time series prediction method

A chaos time series refers to time series of the single variable (which can be multi variables also) acquired through observation and sampling of certain confirmed chaotic dynamic system and the central problem of chaos time series study lies in whether or not it is able to and how to predict the property of the entire dynamic system based upon the observed time series changes and the theoretical basis for this study is reconstruction theory of phase space.

Set the observed chaos time series as $\{x_n \mid n=1,2\ldots,N\}$ with assumption of N =T + L, take the first T data as the samples necessary for

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constituting the model and the last L data as measurement for prediction precision. Adopt the method of phase space reconstruction and then the reconstructed status vector in phase space is as the following:

$$\vec{x}_n = (x_n, x_{n-\tau}, \ldots, x_{n-(m-1)\tau}) \qquad n = T_0 + 1, T_0 + 2, \ldots, T$$

In the formula, T_0 = (m-1)τ, m is called added dimension and τ is delayed time interval. According to Takens conclusion, generally speaking, if m≥2d +1（d stands for dimension of phase space in original dynamic system）, then $\vec{x}_n$ refers to adding of the corresponding one trail in the original dynamic system in R^m.

We will have on R^m one discrete dynamic system F： $R^m \rightarrow R^m$，then：

$$\vec{x}_{n+1} = F(\vec{x}_n)$$

Or we get one function f：$R^m \rightarrow R$，which satisfies：

$$\vec{x}_{n+1} = f(\vec{x}_n) = f(x_n, x_{n-\tau}, \ldots, x_{n-(m-1)\tau})$$

Chaos time series prediction is how to confirm or constitute one similar form $\hat{f}$ （also log F ）of f based upon $\{x_n\}_{n=1}^{T}$. We will know form the chaotic property of system that $\hat{f}$ or $\hat{F}$ should be nonlinear.

Generally speaking, there are two ways of constituting prediction function $\hat{f}$ or $\hat{F}$ as the following:

（1）Field prediction: Use field method for function fitting and prediction and utilize only information of points in the neighborhood of targeted prediction points with main methods of field average prediction method, field linear prediction method, linear interpolation method, field hyper-plane approximation method and max Lyapunov exponent prediction method. Field method is proposed of based upon dynamics characteristics of chaos system, which is able to better reflect evolvement rule of system chaos dynamics in comparison of global method. Therefore it is more practical in most occasions.

（2）Global prediction method: Use global method for function fitting and prediction, namely, use entire data information to approximately predict function and then use its iteration to predict each move in future with main methods of multiple-item type of approximation method, nerve network prediction method, redial basis model prediction method, wavelets nerve network mode and fuzzy nerve network prediction model, etc. Its disadvantage lies in the following aspect: the regular computation becomes complicated if added to high dimension or very complicated $\hat{f}$ or $\hat{F}$.

The global method takes all points in the track as the fitting targets without the consideration of internal characteristics of chaos dynamics. Therefore its prediction precision is not high.

2.1 Max predictable time T

In practical work, time series of vibration signals is apparently random, which might results form chaotic behaviors of rotating mechanical system due to its nonlinearity and certainty. Chaotic motion belongs to the motion, which determines the intrinsic randomness of system and its behavior is extremely dependent on initial conditions. Chaotic system starts from two initial points with adjacency to each other and form two tracks and possibility for difference in short period is little but the difference might be large in enough long time in future. Such kind of "Butterfly Effect "indicates that it is not possible to execute long-term & precise prediction. Nevertheless, the intrinsic certainty of chaotic motion is able to execute precise short-term prediction.

Therefore, it is necessary to solve the problem of predictability at first, that is, max predictable time yardstick, before status prediction. The study of predictable time yardstick of complicated mechanical system is not only meaningful in instructing prediction, but also significant for determining maintenance time and date for great repair.

Max Lyapunov exponent indicates the effective limit for the prediction of future behaviors of the system. Generally speaking, it takes reciprocal value of Lyapunov exponent to represent the max predictable time yardstick because error will be amplified in times if it exceeds this time yardstick, which will cover actual status information.

We can reach the max Lyapunov

exponentλ_1 based upon Wolf method and its improved algorithm, then the max predictable time yardstick is $T = \frac{1}{\lambda_1}$

Definition of max predictable time yardstick T is that the prediction is only precise under the condition of $\triangle t < T$ in the aspect of tract meaning, and it does not indicate that it is unable to predict or unpredictable for long-term prediction if it exceeds T, which means that it can be only used for statistic prediction under the condition of $\triangle t > T$.

1. Use time series to select delayed time τ, and based upon the total amount of observed data sample N, constitute new series of m-dimension space with phase number N_m as $N_m = N - (m-1)\tau$;
2. Take initial phase point X_0 as basic point, select among remaining phase points from point set $\{X_i, i = 1,2,\ldots,N_m\}$ the point X_j that closes to X_0 as endpoint to constitute initial vector, and then the Euclidean distance between X_0 and X_j can be recorded as L(t);
3. Time step is k , $t_1 = t_0 + k$, and we get a new vector when initial vector evolves along with the track forward and then the Euclidean distance between the corresponding basic point and endpoint can be recorded as $L(t_1)$. The increase rate of system linear exponent is recorded as:
$$\lambda = \frac{1}{k}\ln\frac{L(t_1)}{L(t_0)} \quad (1)$$
4. Keep this way until it reaches all phase points and then take the average value of increase rate of all exponents as the max Lyapunov exponent estimation value:
$$\lambda_1 = \frac{1}{N_m}\sum_{i=1}^{m}\frac{1}{k}\ln\frac{L(t_i)}{L(t_{i-1})} \quad (2)$$
5. Add dimension m in sequence and repeat the steps 2, 3 and 4 until the Lyapunov exponent estimation value is converged to a certain fixed value. At that time, the acquired computation result refers to the desired max Lyapunov exponent value.

2.2.2 Specific prediction process

Set reference state as $X(t_n - (m-1)\tau)$, its adjacent state as $X_{nb}(t)$, and corresponding shortest distance as d_{nb} , then:

2.2 Prediction model base upon max Lyapunov exponent

We will see from chaos dynamics theory that the Lyapunov exponent can be used to represent system chaotic behavior and exponent divergence in adjacent tracts in phase space, which depicts the geometrical characteristic of phase volume contraction or expansion in phase space. Therefore, Lyapunov exponent is a good prediction parameter.

2.2.1 Calculate max Lyapunov exponent of time series

$$d_{nb} = \min_i(\|X(t_n - (m-1)\tau) - X(t_i)\|)$$
$$= \|X(t_n - (m-1)\tau - X_{nb}(t)\|, i = 1,2,\ldots,(m-1)$$

Set reference state as $X(t_n - (m-1)\tau)$, which is evolved as $X(t_n - (m-1)\tau + T)$ through advanced prediction time T. Apparently, if $T \le \tau$, then only the last component $X(t_n)$ in $X(t_n - (m-1)\tau + T)$ is unknown and all other m -1 components are known, then:

$$2^{\lambda_1 k} = \frac{\|X(t_n - (m-1)\tau + T) - X_{nb}(t_1 + T)\|}{\|X(t_n - (m-1)\tau - X_{nb}(t_1)\|} \quad (3)$$

λ_1 refers to max Lyapunov exponent value, and formula (3) refers to the max Lyapunov exponent value prediction pattern.

This kind of field method is easy and convenient in calculation with higher precision, which makes it widely applied in chaos time series prediction.

3. Application of prediction method based upon max Lyapunov exponent in rotating machine's state prediction

M max Lyapunov exponents of rolling bearing system in different states belong to positive numbers and vibration signals of rolling bearing belong to chaos time series, which can be predicted according to prediction model of max Lyapunov exponent.

Take vibration signals with severe fault in outer ring as an example to introduce the application of this kind of prediction method in rotating mechanical system. Firstly, we get the system max predictable time through entropy of Kolmogrov as 0.1900 in rolling axis with severe fault in outer ring, then the max predictable time T as: $T = \frac{1}{K} = \frac{1}{0.1900} = 5.2632$. Therefore, the system max predictable time is 5 (point number or step), that is, the prediction is creditable under the condition of T≤5.

Take data of 200 points from time series of rolling bearing's vibration signals for prediction and use the later measured points for reference and comparison. Utilize the above-mentioned prediction pattern to execute 5-step prediction of time series of the vibration signals with result as Table 1. We will see from Table 1 that the prediction in first 3 steps is roughly close to the measure data but the error becomes large from step 4 because severe fault in outer ring of bearing exists, transient impact occurs during the rotating process of bearing, mutation of vibration signals happens, all of which result in large prediction error.

Table1 Prediction result of rolling bearing's vibration signals

Prediction point number	Measured data	Prediction data
1	-0.0418	-0.0313
2	0.0127	0.0109
3	0.0003	0.0002
4	-0.0910	0.0016
5	-0.1109	0.0078

As to normal bearing system, the entropy of Kolmogrov is 0.1809 with max Lyapunov exponent as 2.1342 and max predictable time as 6. We will execute 10-step prediction with the above-mentioned method and the result indicates that errors become larger and larger between prediction data and measure data after step 7 with less and less prediction credibility and the prediction is creditable under the condition of T≤6, which accords with the theory.

According to the above-mentioned analysis, max Lyapunov exponent's prediction model is different from other traditional prediction methods while executing chaos series prediction and the prediction is credible within max predictable time as to chaos signals with relatively slow changes.

Max Lyapunov exponent's prediction model is different from other traditional prediction methods, which is unable to execute precise prediction of mutation of chaos signals but prediction is credible within max predictable time as to chaos signals with relatively slow changes.

4. Conclusion

It introduces max Lyapunov exponent's prediction model to rolling machine's fault diagnosis and explains the max predictable time of chaos time series. At the same time, it points out that is unable to execute precise prediction of mutation of chaos signals while predicting chaos series because max Lyapunov exponent's prediction model is different from other traditional prediction methods.

References

1. Jiang Wanlu, Zhang Shuqing, Wang Yiqun, Fault information diagnosis based upon chaos and wavelets. National Defense Industrial Press, 2005
2. Lv Jinhu, Lu Jun'an, Chen Shihua, Chaos time series analysis and application [M]， Wuhan University Press, 2002
3. Hu Xiaotang, Nonlinear time series prediction study in rotor system. [Academic dissertation for master degree], Changsha: Mechanical-electronic Project Department of National University of Defense Technlogy, 2000
4. Chen Zhe, Feng Tianjin, Zhang Haiyan, Chaos time series analysis based upon wavelets nerve network and reconstruction of phase space [J] . Computer Study and Development, 2001，38(5)：592～596

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